CONNECTION BETWEEN PULSATION SPECTRUM OF A FURANCE FLAME AND THE AIR EXCESS

Yu. F. Maksimenko

UDC 536.46

A dynamic model of the homogeneous reaction in a stream under isothermal conditions is examined. The transmission properties of the model, which are responsible for the connection between the pulsation spectrum of the flame and the coefficient of air excess, are studied.

Flame pulsations in the furnaces of boiler units represent a random process whose characteristics depend on the mode of combustion. The purpose of the present article is to provide an approximate theoretical description of this dependence based on simple assumptions concerning the kinetics of the chemical reaction and structure of the flame which are adopted in the theory of furnace processes [1].

Let us consider the following example of a furnace installation (Fig. 1).

The fuel B and air V enter through a system of burners into the rectangular furnace where the process of flame combustion of the fuel takes place. The flame consists of three zones:

a) The preparation zone 1 where the fuel and air are heated and mixed with recirculation flows of the combustion products;

b) the ignition zone 2 where the flame elements (microvolumes) exist in a state corresponding to their induction periods: the ignition front 4 is the outer boundary of this zone;

c) the combustion zone where the chemical reaction takes place.

Jet flow occurs everywhere except for the preparation zone where turbulent mixing occurs. The jets do not expand. This assumption is valid for those parts of the jets which are located beyond the ignition front. Actually, processes of evaporation or gasification occur near the ignition zone, since they mainly precede combustion. Moreover, the temperature gradients are not great since the temperature field is strongly equalized by radiant heat transport. Therefore thermal expansion of the jets is also insignificant.

Part of the jets returns along the recirculation contours to the base of the flame and penetrates into the preparation zone. It is assumed that a single main prevailing recirculation flow exists, making it possible to neglect the other flows.

Stabilization of the flame is accomplished by the combustion products carried into the preparation zone by the recirculation flows. The mode of combustion is diffusional-kinetic, described on the basis of the assumption that the second-order reactions are isothermal with a constant rate constant. Mixing is absent in the combustion zone.

Let us move to a mathematical description of the process. We will order the elements of the flame. Let us sort the entire set of its elements into subsets for which the degree of depletion of the fuel is the same. It is obvious that these subsets do not intersect and their sum fills the entire flame. Each value of the degree of depletion corresponds to a certain combustion time τ which has elapsed from the moment of ignition of the element. Therefore the set of all the elements of the flame can be ordered with respect to the combustion time, with the subset E_0 corresponding to the zero time and coinciding with the ignition zone.

Let us find the concentrations of components in the elements of the set E_τ. In accordance with the assumptions made we can write the equations of the reaction (to simplify the notation the arguments of the functions will be omitted):

Institute of Fossil Fuels, Moscow. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 26, No. 5, pp. 930-937, May, 1974. Original article submitted May 15, 1973.

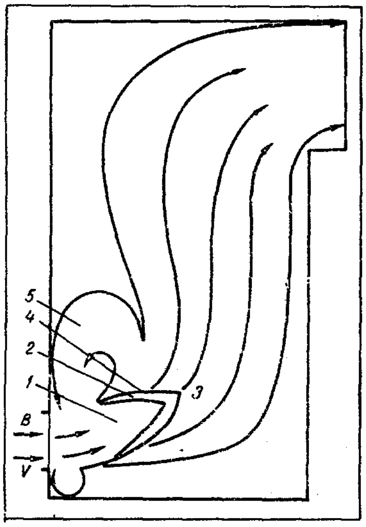

Fig. 1. Structure of flame; 1) preparation zone; 2) ignition zone; 3) combustion zone; 4) ignition front; 5) recirculation flow.

$$\frac{d\xi}{dt} = -K\xi c,$$

$$\frac{dc}{dt} = -\mu K\xi c.$$

Integrating these equations with the condition $c_0 - c = \mu(\xi_0 - \xi)$ (the law of mass action), we find the depletion functions

$$\xi(t, \tau) = \xi_0 \frac{\Delta}{\delta}, \qquad c(t, \tau) = c_0 \frac{\Delta}{\delta} \exp(\tau K \Delta), \tag{1}$$

where

$$\Delta = c_0 - \mu\xi_0, \quad \delta = c_0 \exp(\tau K\Delta) - \mu\xi_0.$$

The equations (1) have meaning for any element of the mixture which has burned for the time τ. Naturally, if an element is considered at the time t and it is known that τ is its combustion time then the moment of ignition must correspond to the value $t-\tau$. Hence it follows that the concentrations of components in the elements of the set E_τ are described by the functions $(t-\tau, \tau)$ and $c(t-\tau, \tau)$ given by Eqs. (1).

Let us write the condition of continuity of the flame. If $\nu(x)$ is the volume of the set x then

$$\nu(dE_\tau) = d\nu(E_\tau) = \frac{d\nu(E_\tau)}{d\tau} d\tau. \tag{2}$$

For nonexpanding streamtubes the function $\chi(\tau) = d\nu(E_\tau)/d\tau$ is constant at all points except for those which correspond to periods of the recirculation. At these points the function $\chi(\tau)$ has a discontinuity since a part of the elements of the flame go outside the limits of the combustion zone into the preparation zone. Since according to the assumptions a single cycle of recirculation is considered, the function $\chi(\tau)$ has one discontinuity. Outside the limits of this discontinuity the function $\chi(\tau)$ does not affect the dynamics of the system and therefore is ignored.

It follows from the assumption of immiscibility that the condition of closure of the flame can be represented in the form

$$E = \bigcup_\tau E_\tau, \quad \tau \in [0, T], \qquad E_{\tau_i} \cap E_{\tau_j} = \varnothing \quad \text{for} \quad \tau_i \neq \tau_j. \tag{3}$$

The equations of material balance for the ignition zone are:

$$\nu_0 \frac{d\xi_0}{dt} = -f_\xi + B + R_\xi, \qquad \nu_0 \frac{dc_0}{dt} = -f_c + V + R_c. \tag{4}$$

The equations of material balance for the combustion zone, assuming that the amount of material in the zone is constant, are:

$$f_\xi - \psi_\xi - Su\xi(t-T, T) - R_\xi = 0, \qquad f_c - \psi_c - Suc(t-T, T) - R_c = 0. \tag{5}$$

Eliminating the function $f_\xi - R_\xi$ and $f_c - R_c$ from Eqs. (4) and (5) we obtain equations describing the variations in the initial concentrations:

$$\nu_0 \frac{d\xi_0}{dt} = -\psi_\xi - Su\xi(t-T, T) + B, \qquad \nu_0 \frac{dc_0}{dt} = -\psi_c - Suc(t-T, T) + V. \tag{6}$$

Then it is necessary to calculate the depletion rate ψ_ξ and ψ_c. For an arbitrary point of space in the flame the combustion rate with respect to the fuel is equal to the derivative $-(\partial/\partial\tau)\xi(t-\tau, \tau)$. For the

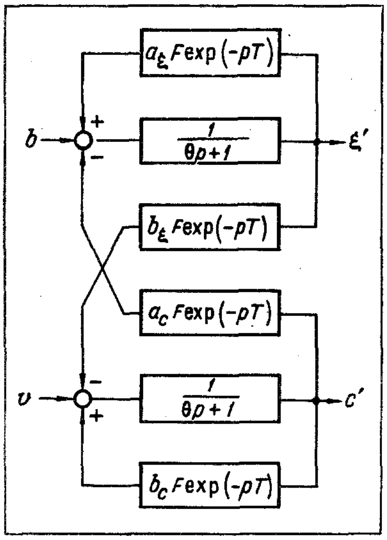

Fig. 2

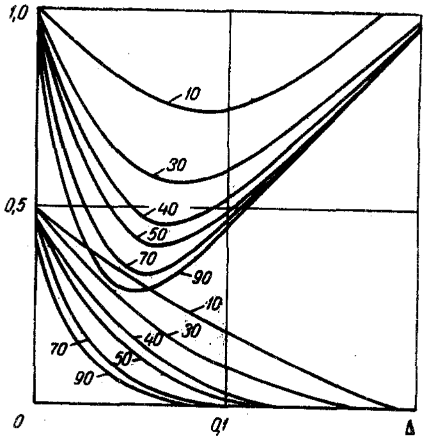

Fig. 3

Fig. 2. Block diagram of linear transformations of disturbed process of combustion: b: disturbance in fuel; v: disturbance in oxygen.

Fig. 3. Dependence of parameters of system on air excess (the signs are taken into account in Fig. 2): upper family: function $(a_\xi + b_c)$; lower family: function $(a_c b_\xi + a_\xi b_c)$. The numbers show the value of the parameter TK, m^3/kg, and the parameters μ and c_0 are taken as: $\mu = 1.44$; $c_0 = 0.21$.

flame as a whole the fuel combustion rate is equal to the integral of this value taken over the entire volume of the flame. Using the concept of the Lebesgue—Stieltjes integral [2] and the condition (3) we obtain

$$\psi_\xi(t) = -\int_E \frac{\partial \xi(t-\tau, \tau)}{\partial \tau} \nu(dE_\tau).$$

Using Eq. (2) we can change to the Riemann integral

$$\psi_\xi(t) = -\int_0^T \frac{\partial \xi(t-\tau, \tau)}{\partial \tau} \chi(\tau)\, d\tau.$$

Repeating these operations for the function ψ_c we can reduce Eqs. (6) to form

$$\begin{aligned} v_0 \frac{d\xi_0}{dt} &= \int_0^T \frac{\partial \xi(t-\tau, \tau)}{\partial \tau} \chi(\tau)\, d\tau - Su\xi(t-T, T) + B, \\ v_0 \frac{dc_0}{dt} &= \int_0^T \frac{\partial c(t-\tau, \tau)}{\partial \tau} \chi(\tau)\, d\tau - Suc(t-T, T) + V. \end{aligned} \tag{7}$$

Equations (7) can be solved approximately with respect to ξ_0 and c_0 for small disturbances. As was shown above, the function $\chi(\tau)$ is constant, and since its integral must be equal to the volume of the flame, $\chi(\tau) = U/T$.

Substituting Eq. (1) into (7) we obtain

$$\begin{aligned} v_0 \frac{d\xi_0}{dt} &= -\frac{U}{T}\xi_0 + \frac{U}{T}\left(\xi_0 \frac{\Delta}{\delta}\right)_T - Su\left(\xi_0 \frac{\Delta}{\delta}\right)_T + B, \\ v_0 \frac{dc_0}{dt} &= -\frac{U}{T}c_0 + \frac{U}{T}\left(c_0 \frac{\Delta}{\delta}\exp(TK\Delta)\right)_T \\ &\quad - Su\left(c_0 \frac{\Delta}{\delta}\exp(TK\Delta)\right)_T + V. \end{aligned} \tag{8}$$

Let us examine the properties of these equations in the vicinity of the equilibrium state ξ_0^*, c_0^* determined by the conditions

TABLE 1. Limiting Values of Parameters

	$\Delta=0, \quad x=C_0TK$
a_c	$\frac{x^2}{2\mu(1+x)^2}-\frac{x}{\mu(1+x)}$
a_ξ	$\frac{x^2}{2(1+x)^2}+\frac{1}{1+x}$
b_c	$\frac{x^2}{2(1+x)^2}+\frac{1}{1+x}$
b_ξ	$-\frac{\mu x^2}{2(1+x)^2}-\frac{\mu x}{(1+x)^2}$

$$\frac{d\xi_0}{dt}=0, \quad \frac{dc_0}{dt}=0.$$

Expanding the nonlinear terms in a power series, we obtain from Eqs. (8) linearized equations of the disturbed motion of the system which, after undergoing a Laplace transformation, take the form

$$\begin{aligned}(\theta p+1)\xi' &= Fa_c\exp(-pT)c' + Fa_\xi\exp(-pT)\xi',\\ (\theta p+1)c' &= Fb_c\exp(-pT)c' + Fb_\xi\exp(-pT)\xi',\end{aligned} \tag{9}$$

where $F = (1-SuT/U)$, $\theta = \nu_0T/U$.

The coefficients of Eqs. (9) are equal to the following derivatives calculated at the point (ξ_0^*, c_0^*):

$$\begin{aligned}a_c &= \frac{\partial}{\partial c_0}\left(\xi_0\frac{\Delta}{\delta}\right), \quad a_\xi = \frac{\partial}{\partial \xi_0}\left(\xi_0\frac{\Delta}{\delta}\right),\\ b_c &= \frac{\partial}{\partial c_0}\left(c_0\frac{\Delta}{\delta}\exp(TK\Delta)\right), \quad b_\xi = \frac{\partial}{\partial \xi_0}\left(c_0\frac{\Delta}{\delta}\exp(TK\Delta)\right).\end{aligned} \tag{10}$$

Equations (9) correspond to the block diagram illustrated in Fig. 2, from which it is seen that the combustion process with respect to the disturbances acting on it represents a dynamic system with delayed positive feedbacks. If random pulsations in concentration arise in the process of ignition, the spectrum of these pulsations, transformed by the dynamic system of the flame, will depend on the mode of combustion. So-called signal filtration takes place. The law according to which this filtration is accomplished is known: the spectral density of the output signal is equal to the product to the spectral density of the disturbance times the square of the modulus of the transmission function [3].

Let us investigate how the transmission properties of the flame are connected with the mode of combustion. Using the topological method [4] it is easy to find transmission functions directly from the diagram of Fig. 2. We write the characteristic polynomial as

$$L(p) = 1-(a_\xi+b_c)F\frac{\exp(-pT)}{\theta p+1}-(a_cb_\xi+a_\xi b_c)F^2\frac{\exp(-2pT)}{(\theta p+1)^2}. \tag{11}$$

Only the coefficients a_ξ, b_c, a_c, and b_ξ depend on the mode of combustion. The coefficient F is determined by the geometry of the furnace and the recirculation of the flame. If recirculation is absent then $F = 0$. Actually, the value $\chi(\tau) = U/T$ is equal to the volumetric flow rate of material passing through the ignition front while Su equals the volumetric flow rate of the combustion products. If there is no recirculation then $U/T = Su$ and $F = 0$. The limiting value of F is equal to unity.

Let us clarify how the values given by Eqs. (10) depend on the coefficient of air excess. For the purpose of simplification it is convenient to estimate the air excess by the value Δ which is connected with it by a monotonic dependence.

The dependences of the parameters of the transmission functions calculated in accordance with Eqs. (10) are presented in Fig. 3, while their limiting values are given in Table 1. Let us now turn to the analysis of the dependence of the transmission functions on the air excess.

From a qualitative point of view it will be sufficient to confine the study to one transmission function. Let us discuss some of the simplifications adopted in the analysis.

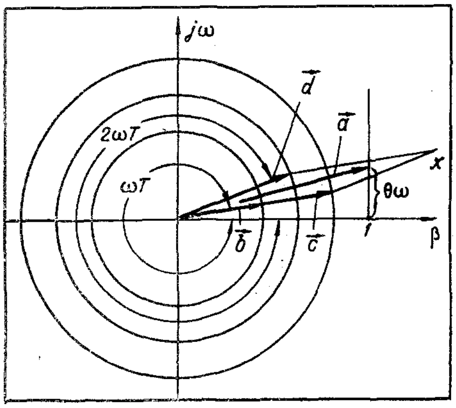

Fig. 4. Vector diagram clarifying the dependence of the pulsation spectrum on the air excess.

First, we will calculate the modulus of the transmission function at a frequency $\omega > 0$ for which the following inequality is satisfied: $\theta\omega \ll 1$. Physically this means that the frequency ω is not damped by the inertial properties of the system.

Second, let us set $F \gg 0$. This will mean that recirculation is significant. In the following constructions we will take $F = 1$, which does not affect the qualitative result.

With these assumptions the transition function will equal

$$W(j\omega)=\frac{\xi'}{b}\cong\frac{\theta\omega j+1-b_c\exp(-j\omega T)}{\theta\omega j+1-(a_\xi+b_c)\exp(-j\omega T)-(a_c b_\xi+a_\xi b_c)\exp(-2j\omega T)}. \quad (12)$$

The modulus of $W(j\omega)$ is easily estimated graphically. Let us consider the complex plane (β, $j\omega$) (Fig. 4). The straight line passing vertically through the point (1, 0) is the geometrical locus of the ends of the vector $\theta\omega j + 1$, designated as **a**. The circles with radius vectors **b**, **c**, and **d** are the functions $b_c \exp \times (-j\omega T)$, $(a_\xi + b_c)\exp(-j\omega T)$ and $(a_c b_\xi + a_\xi b_c)\exp(-2j\omega T)$, respectively. According to Eq. (12) the modulus of the transmission function will equal

$$|W| = \frac{|\mathbf{a}-\mathbf{b}|}{|\mathbf{a}-(\mathbf{c}+\mathbf{d})|}. \quad (13)$$

It is seen from Eq. (13) and Fig. 4 that as the radii of the circles decrease the vector **c** + **d** whose end lies at the point x approaches the vector **a**. Their difference becomes very small.

Hence it follows that the amplitude of the random pulsations is an extremal function of the air excess. This conclusion is valid for some frequency ω which in accordance with Fig. 4 satisfies the inequality

$$\operatorname{arctg}\theta\omega > (2\pi-\omega T) > 0.$$

Obviously, an analogous relationship will be observed with respect to any physical value connected with the concentrations. Experimental confirmation of this fact can be found in [5].

The problem examined can have important technical applications in the following directions:

1. The determination of the aerodynamic structure of a flame from its pulsation spectra. One can determine the periods of the recirculation cycles by finding out in which frequency bands of the spectrum the extrema are observed.
2. The optimum control of the combustion process. Each extremum of the pulsations corresponds to some air excess, with the extremal air excesses being smaller for long contours (see Fig. 3) than for short ones. By installing several probes which pick up the pulsations of different cycles one can obtain information on the current value of the air excess. The obtaining of such information is a necessary condition for optimization of the process.

NOTATION

B	is the weight flow rate of fuel;
V	is the weight flow rate of oxygen;
T	is the time of movement of mixture through recirculation cycle;
t	is the current time;
τ	is the combustion time of a flame elements;
$\xi_0(t)$ and $c_0(t)$	are the concentrations of fuel and oxygen in ignition zone;
$\xi(t, \tau)$ and $c(t, \tau)$	are the concentrations of fuel and oxygen in that element of the combustion zone which ignited at the moment t and has burned for the time τ, where $\xi(t, 0) = \xi_0(t)$ and $c(t, 0) = c_0(t)$;
K	is the rate constant of combustion;
μ	is the stoichiometric coefficient;
E	is the set of all flame elements;
E_τ	is the subset of those flame elements which have burned for the time τ;
ν_0	is the average volume of ignition zone;
$f_\xi(t)$	is the ignition rate of fuel by weight;

$f_c(t)$	is the rate of supply of oxygen to ignition reaction by weight;
$\psi_\xi(t)$ and $\psi_c(t)$	are the rates of consumption of fuel and oxygen in combustion process by weight;
Su	is the product of area of output cross section of flame times the gas discharge velocity;
$R_\xi(t)$ and $R_c(t)$	are the amounts of fuel and oxygen carried into preparation zone by recirculation flows per unit time;
U	is the flame volume;
p	is the parameter of Laplace transform;
ω	is the frequency.

Subscripts

' indicates a deviation from the equilibrium state;
* indicates the equilibrium state;
T indicates a delay by the amount T.

LITERATURE CITED

1. G. F. Knorre, Furnace Processes [in Russian], Gosénergoizdat, Moscow—Leningrad (1951).
2. V. I. Smirnov, A Course in Higher Mathematics [in Russian], Vol. 5, Fizmatgiz, Moscow (1959).
3. V. V. Solodovnikov, Introduction to the Statistical Dynamics of Automatic Control Systems [in Russian], Gostekhizdat, Moscow—Leningrad (1952).
4. L. A. Bessonov, Linear Electrical Circuits [in Russian], Vysshaya Shkola, Moscow (1968).
5. Yu. F. Maksimenko, Yu. V. Ivanov, and M. V. Kleiman, Pribory i Sistemy Upravleniya, No. 2, 29-30 (1971).